

BLACK BRANE WORLD SCENARIOS *

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We consider a brane world residing in the interior region inside the horizon of extreme black branes. In this picture, the size of the horizon can be interpreted as the compactification size. The large mass hierarchy is simply translated into the large horizon size, which is provided by the magnitude of charges carried by the black branes. Hence, the macroscopic compactification size is a quantity calculable from the microscopic theory which has only one physical scale, and its stabilization is guaranteed from the charge conservation.

In this talk, we will pay attention to the interior region bounded by a degenerated horizon of extreme black branes. The interior region possesses all of the features needed for large extra dimension (ADD)¹ as well as Randall-Sundrum (RS) scenarios² provided that the interior region is regular. Since the interior region has finite volume while it asymptotes to the infinitely long AdS throat, the central region inside the near horizon region acts like a domain wall embedded in anti-de Sitter space (AdS_5) like the RS brane. The massless 4D graviton is clearly localized to the central region, reproducing the correct 4D Newtonian gravity.

The size of the horizon can be interpreted as the compactification size in that the four-dimensional Planck scale M_{pl} is determined by the fundamental scale M_* of the higher-dimensional theory and the size of the horizon r_H via the familiar relation $M_{pl}^2 \sim M_*^{2+d} r_H^d$ and the effective gravity on a 3-brane residing in the central region has a transition from the four-dimensional to the higher-dimensional gravity around distances of the size of the horizon. This scenario gives a natural explanation of the large mass hierarchy between the four-dimensional Planck scale M_{pl} and the weak scale m_{EW} . The large mass hierarchy is translated into the large size of the horizon. The large size of the horizon, *i.e.*, the small compactification scale is provided with the large magnitude of charge carried by the black branes, that is, winding number or R-R charge. Hence, the macroscopic compactification size now is a quantity calculated from the microscopic theory and its stabilization is guaranteed from the charge conservation.

We will consider two types of black brane solutions. Firstly, we will show

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a global black brane solution^{8,9} which is a black hole like defect solution to a scalar theory with global $O(d)$ symmetry coupled to higher dimensional gravity and is a p -dimensional extended object surrounded by a degenerated horizon. This solution is perfectly regular everywhere. Secondly, we will consider the supergravity solution of the D3-brane. Its interior region interpolates between the singularity and the near horizon region. When the singularity is smoothed out by stringy effects, the graviton is localized in the central region. Further, even in the existence the gravity could be trapped, under the assumption of the unitary boundary condition for the graviton states¹⁰.

The scalar theory with global $O(d)$ symmetry coupled to D -dimensional gravity, of which potential has minimum on the $(d-1)$ -sphere with radius v^2 allows a black hole like extended defect solution, which we will call as ‘global black p -brane’. The global black p -brane is a p -dimensional extended object surrounded by a degenerated horizon. This object is very similar to the black p -brane solutions of supergravity and string theories. However, this solution is perfectly regular everywhere, that is, either inside or outside of the horizon.

Here, we will not write down the Einstein’s equations of motion and the scalar field equation of motion, and we will skip the procedure to find the global black brane solution. These are treated in detail in Refs.[8,9]. We will begin just with a brief description for the spacetime of the global black brane. We introduce the following Schwartzschild-type metric ansatz:

$$ds^2 = e^{2N(r)} B(r) \bar{g}_{\mu\nu}(x) dx^\mu dx^\nu + \frac{dr^2}{B(r)} + r^2 d\Omega_{d-1}^2, \quad (1)$$

where $\bar{g}_{\mu\nu}(x)$ is a general Ricci-flat metric on the brane, which satisfies $(p+1)D$ Einstein equations $\bar{R}_{\mu\nu}(\bar{g}) = 0$. We have looked into the behavior of solutions at a few regions because it seems almost impossible to find exact analytic solutions. Outside the core, the behavior of metric functions is controlled by the ratio between the scalar field energy density ($8\pi G_D/r^2$) and the cosmological constant ($|\Lambda|$). In the far region where the cosmological constant dominates over the field energy density. The metric (1) has the form of

$$ds^2 \approx B_\infty r^2 \bar{g}_{\mu\nu}(x) dx^\mu dx^\nu + \frac{dr^2}{B_\infty r^2} + r^2 d\Omega_{d-1}^2, \quad (2)$$

where $B_\infty \equiv 2|\Lambda|/(p+d)(p+d-1)$. This corresponds to D -dimensional anti-de Sitter space (AdS_D) and can be changed to the form found in Refs.[3,4] using the proper radial distance $\chi(\equiv \int^r dr'/\sqrt{B(r')})$ as

$$ds^2 \approx e^{2\sqrt{B_\infty}\chi} \bar{g}_{\mu\nu}(x) dx^\mu dx^\nu + d\chi^2 + e^{2\sqrt{B_\infty}\chi} d\Omega_{d-1}^2. \quad (3)$$

The horizon occurs in a region where the field energy density and the cosmological constant are comparable, and the spacetime is approximated by

$$ds^2 \approx B_H [\sigma(r - r_H)]^{2(1-\alpha)} \bar{g}_{\mu\nu}(x) dx^\mu dx^\nu + \frac{dr^2}{B_H(r - r_H)^2} + r_H^2 d\Omega_{d-1}^2, \quad (4)$$

where σ is -1 for the interior region ($r < r_H$) and $+1$ for the exterior region ($r > r_H$) and the curvature scale of this spacetime and the horizon size are given by

$$k^2 \equiv B_H(1 - \alpha)^2 = \frac{2|\Lambda|}{(p+1)(p+d-1)}, \quad (5)$$

$$r_H^2 = \frac{(p+d-1)(8\pi G_D - d + 2)}{2|\Lambda|}. \quad (6)$$

It is easy to see relation between the near horizon solution and the cigar-like warped spacetime obtained in Refs.[3,4]. Introducing a new radial coordinate $\chi(> 0)$ defined by $\exp(-k\chi) \equiv \sqrt{B_H} [\sigma(r - r_H)]^{1-\alpha}$, the metric Eq.(4) is rewritten as

$$ds^2 \approx \exp(-2k\chi) \bar{g}_{\mu\nu}(x) dx^\mu dx^\nu + d\chi^2 + r_H^2 d\Omega_{d-1}^2. \quad (7)$$

Hence, among solutions obtained in Refs.[3,4], the meaningful solution with respect to the Randall-Sundrum scenario can naturally be interpreted as the near horizon geometry of a black p -brane. The two solutions (2) and (7), which seemed to be disjointed in Refs.[3,4], can be matched to each other. On the other hand, in the region between the brane core and the near horizon where the cosmological constant is negligible compared to the field energy density (if $\Lambda \ll M_*$, this region overwhelm the near horizon region), the shape of spacetime is largely dependent on dimension of transverse space. When $d = 2$, the geometry of this region will resemble the Cohen-Kaplan spacetime⁵. When $d \geq 3$, the solution in this region will be close to the global monopole metric⁴. While we have not cleared up whether the solutions obtained in separated regions can be connected mutually or not, numerical works explicitly show that such black brane solutions exist.

The interior region is of interest, because it possesses all of the features needed for RS scenario, that is, it asymptotes to AdS space and its volume is finite and concentrated near the brane.

In the RS scenario, the necessary condition to have the usual Newtonian gravity on the brane is the existence of a massless 4D graviton localized on the brane. From now on, we will consider only $p = 3$. The existence of the massless graviton state is evident because Einstein's equations of motion

always allows solutions with a general Ricci flat metric $\bar{g}_{\mu\nu}(x)$ and the massless 4D graviton is simply the usual gravitational wave solution of linearized 4D vacuum Einstein equation. And the boundness of the massless graviton is equivalent to the condition that the 4D Planck scale M_{pl} is finite. Examination of the 4D effective action yields the brane world Planck scale:

$$M_{pl}^2 = M_*^{d+2} \int dz^d \sqrt{g_D} g^{00} \sim M_*^{2+d} r_H^d. \quad (8)$$

Thus, it is now clear that a massless state of the 4D graviton is bounded. The relation (8) tells that the 4D Planck scale is determined from the fundamental scale M_* and the horizon size r_H via the familiar relation from usual Kaluza-Klein theories. This implies that the horizon size r_H can be interpreted as the effective size of d compact extra dimensions, even though the interior region infinitely extends. From the point of view of one who lives in infinitely extended higher dimensional spacetime, this seems indeed a correct interpretation, clearly the interior region takes only a finite part with volume $\sim r_H^d$ of the infinite transverse space and the apparent infinite extent of the interior region is simply a result of the warping of the finite region of extra space by the gravity of the brane itself. This interpretation could be cleared up through a complete analysis of the effective 4D gravity.

In order to see the effective gravity on the brane, we introduce the perturbations by replacing $\bar{g}_{\mu\nu}(x)$ with $\eta_{\mu\nu} + h_{\mu\nu}(x, z)$ in Eq.(1). Imposing the RS gauge on $h_{\mu\nu}$, we can easily find the linearized field equation for $h_{\mu\nu}$. Making a change of variables $\xi = \int^r \sqrt{-g^{00}(r')g_{rr}(r')} dr'$, $\tilde{h}_{\mu\nu} = K h_{\mu\nu}$ and separating $\tilde{h}(\xi, \Omega) = \epsilon_{\mu\nu} e^{ip \cdot x} R_{m\ell}(\xi) Y_\ell(\Omega)$, the linearized field equation can always be written into the form of an analog non-relativistic Schrödinger equation:

$$\left[-\frac{\partial^2}{\partial \xi^2} + V_{eff}(\xi) \right] R_{m\ell}(\xi) = m^2 R_{m\ell}(\xi), \quad (9)$$

$$\text{with } V_{eff}(\xi) \equiv \frac{K''(\xi)}{K(\xi)} + \ell(\ell + d - 2) \frac{g_{00}(\xi)}{r(\xi)^2}, \quad (10)$$

where $K \equiv r^{(d-1)/2} g_{00}^{3/4}$ and $Y_\ell(\Omega)$ is a d -dimensional spherical harmonics. Here $\epsilon_{\mu\nu}$ is a constant polarization tensor and m ($= \sqrt{p \cdot p}$) is the mass of continuum modes. The zero-mode wave function, with $m = \ell = 0$, is easily identified as $R_{00}(\xi) = K(\xi)$. All of the important physics follows from a qualitative analysis of the effective potential.

The Newtonian potential generated by a point source of mass m^* localized

on the brane then is

$$U(|\vec{x}|) = -G_4 \frac{m^*}{|\vec{x}|} - \frac{m^*}{M_*^{d+2}} \sum_{\ell} \int_{m \neq 0} dm m^{\delta} |R_{m\ell}(0)|^2 \frac{e^{-m|\vec{x}|}}{|\vec{x}|}. \quad (11)$$

m^{δ} is contribution from more than one extra dimensions for measures of relevant density of states. If the extra dimensions are noncompactified, then $\delta = d - 1$, simply. The factor of m^{d-1} is just the d -dimensional plane wave continuum density of states (up to a constant angular factor). In our case, $d - 1$ extra dimensions are compactified to S^{d-1} with radius r_H . The modes with small ℓ behave as plane waves only in the radial direction, while the modes with large ℓ do in the full extra dimensions. Thus, δ will depend on ℓ , that is, $\delta = 0$ for modes with small ℓ but $\delta = d - 1$ for sufficiently large ℓ .

In the original warped bulk model of Randall-Sundrum type scenario, the bulk cosmological constant Λ had always been identified with the fundamental scale for naive naturalness reasons. However, given our ignorance regarding the cosmological constant problem, we do not feel any strong prejudice forcing Λ to be of the order of the fundamental scale M_*^2 . We will simply treat Λ as a parameter; we know only that r_H must be smaller than $\sim 1\text{mm}$ from the present-day gravity measurements if it should be interpreted as a compactification size. We will consider two limiting values of Λ ; $|\Lambda| \sim M_*^2 \sim M_{pl}^2$ and $|\Lambda| \sim 10^{6-60/d} \text{GeV}^2 \ll M_*^2 \sim m_{EW}^2$. We will call the first limit as Randall-Sundrum (RS) limit and the second limit as large extra dimension (ADD) limit.

In the RS limit, the interior region is well approximated by the near horizon geometry Eq.(7) because both the core radius $r_c \sim (\sqrt{\lambda}v)^{-1/2}$ and the horizon size r_H are of the order of the fundamental scale and so $8\pi G_D/r^2$ and $|\Lambda|$ are comparable in whole interior region. In this regime, since both curvature scales of AdS_5 and S^{d-1} are of the order of the fundamental scale, the extra space effectively reduces to one-dimensional space because one who lives in the bulk could not observe the extra S^{d-1} through low-energy processes under the fundamental scale. Then the 3-brane looks like a one-sided RS-brane embedded in a AdS_5 bulk spacetime. Hence the physics on the brane will be nearly the same with that of RS scenario. A difference arises due to massive KK modes with $\ell \neq 0$ living in the AdS_5 . However, this correction is strongly suppressed due to the repulsive centrifugal potential. Thus, the low-energy physics on the brane are imperceptibly different from those in the RS scenario.

In the ADD limit, the phenomenologically acceptable size of the hori-

zon (cosmological constant) is $r_H \sim 10^{30/d-17}\text{cm}$ ($|\Lambda| \sim 10^{6-60/d}\text{GeV}^2$) with $M_* \sim M_{EW} \sim 10^3\text{GeV}$ and $M_{pl} \sim 10^{18}\text{GeV}$. Then the AdS region takes extremely tiny portion of the transverse space and the intermediate region, in which $8\pi G_D/r^2 \gg |\Lambda|$, between the core and the AdS region occupy most volume of transverse space.

The geometry of the intermediate region can be approximated by that of Cohen-Kaplan solution⁵ when $d = 2$ and those of global monopole spacetime⁴ when $d \geq 3$. It is easy to explicitly calculate the potential Eq.(10) using the Cohen-Kaplan and the global monopole metric. The explicit calculations show that, when $d = 2, 3$, the potential is attractive or zero in the intermediate region. However, it is not needed to specify details of the the shape of the central region of the potential, because the potential is not repulsive in the intermediate region and the central region is localized within the the AdS length scale k^{-1} . Then the leading corrections to the long-range gravitational potential are in fact identical to those in the RS limit, as shown in Ref.[7]. The short distance gravity is dominated by continuum modes with $m \geq k$ and $\ell \gg 1$ and $(4 + d)\text{D}$ gravity appears at distances $|\vec{x}| \ll r_H$. Hence, in this regime, the brane world gravity behaves like that of RS limit except that it experience a transition from $\sim 1/|\vec{x}|$ to $\sim 1/|\vec{x}|^{d+1}$ around the distances of the size of the horizon, which is now much bigger than the weak scale, *i.e.*, $r_H \gg m_{EW}$. Hence, the behavior of the gravity on the brane is exactly what we would expect by interpreting the size of the horizon as a ‘‘compactification radius’’.

When $d \geq 4$, the physics will be quite different from that of when $d = 2, 3$ due to the repulsive potential in the intermediate region. Since the hight of the potential is $V_{eff}(\sim r_c) \sim M_{EW}^2$, the continuum modes with mass $m \ll M_{EW}$ have suppressed wave functions near the core and so, for distances $|\vec{x}| \gg M_{EW}^{-1}$, the gravity behaves as in RS limit and is nearly four-dimensional. On the other hand, for distances $|\vec{x}| \ll m_{EW}$, the gravity is D -dimensional. This regime is phenomenologically interesting because the physics on the brane seems very distinct from both the RS and the ADD scenarios, in that the gravity is maintained to be four-dimensional down to distances of m_{EW}^{-1} , despite introducing the large extra dimensions to obtain the 4D Planck scale. Furthermore, there are no light moduli fields associated with the large size of S^{d-1} because they are strongly suppressed near the 3-brane.

In the ADD limit, we have obtained not only the phenomenology of theories with large extra dimensions for $d = 2, 3$ but also somewhat distinct one for $d \geq 4$. In both cases, the hierarchy problem could be resolved via the large

horizon size r_H in the sense of the conventional large extra dimension scenarios. However, there still remains a hierarchy between M_{EW} and $|\Lambda|$, because such large size of the horizon is provided by only the tiny bulk cosmological constant. This hierarchy could be stable in the sense that small changes of $|\Lambda|$ have small effects to the physics on the brane as expected in ADD scenarios. An unpleasant point is the fact that $|\Lambda|$ is a Lagrangian parameter that is not free from the radiative correction due to various bulk fields and needs to be stabilized by a symmetry, such as supersymmetry. In absence of a clear stabilization mechanism, it however seems natural $|\Lambda|$ to be of the order of the fundamental scale. From this aspect, the Randall-Sundrum limit only seems to be natural.

At this point, an immediate question is whether the such large size of the horizon (or equivalently the small compactification scale) can be calculated in the theory that has only one physical scale of the order of the weak scale. The answer seems to be positive. Usually the size of the horizon of an extreme black brane is determined by the magnitude of charge (equivalently mass) carried by the black brane. It does not seem uncommon to have solutions with huge magnitude of charge; the world around us abounds with solutions that has much larger charge than the electron's.

Up to now, we have considered only brane solutions with unit winding number. This was because the conditions of spherical symmetry and regularity at the origin allows only solutions with unit winding number when $d \geq 3$. While cylindrically symmetric solutions with arbitrarily large winding number n are allowed when $d = 2$. The solutions with winding number n can easily be obtained simply replacing $8\pi G_D$ with $8\pi G_D n^2$ in the solution with unit winding number for $d = 2$. Then the radius of the horizon is given from Eq.(6) in terms of unrescaled parameters by $r_H^2 = 16\pi v^2 n^2 / M_*^4 |\Lambda|$. We assume all scales to be in the same order with the fundamental scale, that is, intrinsically there exist only one physical scale. Then the horizon size is given by $r_H \sim n M_*^{-1}$ simply. Putting $M_* \sim m_{EW} \sim 10^3 \text{ GeV}$ and demanding that r_H and n are chosen to reproduce the observed four-dimensional Planck scale $M_{pl} \sim 10^{18} \text{ GeV}$ yields $n \sim 10^{15}$. Hence, the global black brane solution with large winding number of $n \sim 10^{15}$ seems to provide a dynamical determination of the hierarchy between the four-dimensional Planck scale and the weak scale without requiring any additional small scale from the theory that has intrinsically only one physical scale. Furthermore, the such large compactification size is now stabilized via the charge conservation law.

The interior region of the D3-brane interpolates between the singularity and the AdS throat. The similar analysis of the graviton states to that for

the global black brane shows that even in the existence of the singularity the massless graviton can be localized in the central region of the interior region under the unitary boundary condition^{5,6}. However, since we do not have a detailed understanding of the singularity, we will not be able to make any rigorous claim whether such boundary conditions are what we want. Thus, we will only assume that the singularity is smoothed out by the true short-distance theory of gravity, namely, string theory. Essentially the source for the R-R field strength is sitting at the singularity. Since the only known source for R-R 5-form field strength is D3-brane, we are naturally expected to see a stack of D3-branes when we probe the singularity with energy over the string scale.

The analog Schrödinger potential resembles with that for the global black brane with $d = 2$. And so the gravity on a 3-brane residing in the central region behaves like what we would expect by interpreting the horizon size r_H as a compactification radius. Since the effective Planck scale is determined via the relation $M_{pl}^2 = r_H^6 / (24\pi^3 g_s^2 l_s^8)$ and the size of the horizon is given by $r_H^4 = 4\pi l_s^4 g_s N$, the hierarchy between the string scale l_s^{-1} and the four-dimensional Planck scale M_{pl} can be provided with the R-R charge of the amount of $N = \left(2\pi^{3/2} l_s^2 g_s^{1/2} M_{pl}^2\right)^{2/3}$. If we naively assume that the string scale is of the order of the weak scale, *i.e.*, $l_s \sim m_{EW}$, then the amount of the R-R charge needed to generate the hierarchy is $N \sim 10^{22}$ and the size of the horizon is $\sim 10^{-12}$ cm.

Perhaps the deepest consequence of the above picture is that it gives a natural explanation of the large mass hierarchy and gives a good reason of why string theory would necessarily choose such type of compactification geometry. The large mass hierarchy is translated into the large size of the horizon. The large size of the horizon, *i.e.*, the large compactification size can be calculated in string theory that has only one physical scale l_s and is determined by the amount of the R-R charge N carried by the D3-brane. Furthermore, the stabilization of such large compactification size is strictly supported by the conserved D3-brane charges.

We don't need to compactify whole spacetime introducing the compactification manifold. The geometry needed for the Randall-Sundrum type brane world can be obtained simply from the noncompact ten-dimensional spacetime by means of formation of a large cluster of D3-branes.

Let us conclude with some discussions. The above picture may provide new perspectives on problems associated to the brane world scenarios. Both non-Ricci flat metric $\hat{g}_{\mu\nu}(x)$ and metric dependent on the extra dimensional

coordinates z^i , *i.e.*, $\hat{g}_{\mu\nu}(x, z) \sim \eta_{\mu\nu} + h_{\mu\nu}(x, z)$, correspond to excitations upon the extremal black brane background, which is the ground state of the black p -brane. That is, in the existence of such excitations the black brane becomes now non-extremal and its Hawking temperature is not zero. Thus, such excitations will be diluted through Hawking radiation. First, the continuum modes are one of such excitations. The analysis on the singular behavior of the continuum modes at the horizon¹¹ should be reexamined because it has been done on the rigid AdS background, while in the existence of such excitation we lose the AdS background. Second, the Poincaré invariance in the longitudinal direction will persist even in the presence of the quantum corrections to the brane tension because the quantum corrections also will be diluted, so that no 4D cosmological constant is generated. Third, this picture provides a new physical mechanism that can solve the cosmological flatness problem. Even though our world brane was highly bent initially, the bending should have been diluted as the black brane Hawking radiates and evolves toward the extremal one. Since the entropy density of our universe is minutely small, our world brane seems to be embedded in the interior region of very near-extremal black brane. Finally, the another flatness problem¹² associated with the approximate Lorentz invariance in the longitudinal direction also can be resolved because the bulk curvature will be diluted via Hawking process.

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